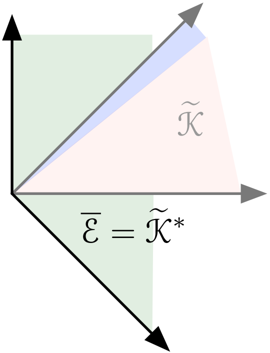
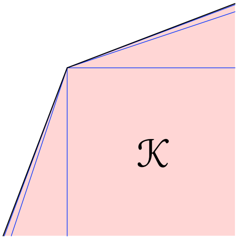
Strings, geometry, and the swampland at the boundaries



Ben Heidenreich
(UMass Amherst)

BH, Rudelius, 2207.xxxxx



BH, Rudelius, 2007.07892
Alim, BH, Rudelius, 2108.08309
Etheredge, BH, Kaya, Qiu, Rudelius, 2206.04063
Gendler, BH, McAllister, Moritz, Rudelius, 22xx.xxxxx

String Pheno 2022, Parallel Session Talk

Swampland program says a lot about moduli space boundaries:

- Swampland Distance Conjecture
- Emergent String Conjecture

Indirectly:

- No global symmetries
- Weak Gravity Conjecture
- Tameness conjecture(?)

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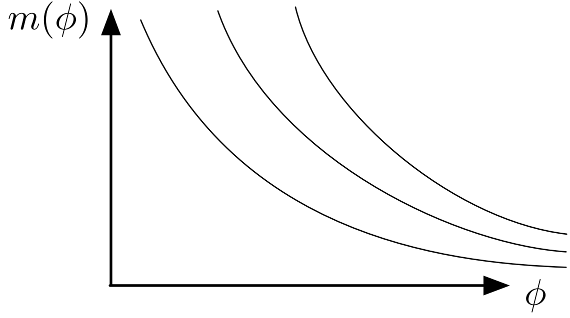
Swampland Distance Conjecture

(Ooguri-Vafa '06)

Infinite distance limits are accompanied by infinite towers of particles become exponentially light



$$m_{\text{tower}} \propto e^{-\kappa_d \lambda \phi}$$

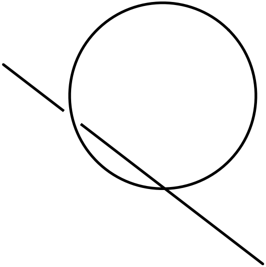
$$\lambda_{\text{lightest}} \geq \frac{1}{\sqrt{d-2}}$$

(Etheredge, BH, Kaya, Qiu, Rudelius '22)

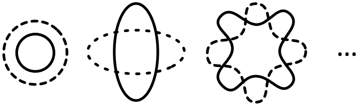
Emergent String Conjecture

(Lee, Lerche, Weigand '19)

Each infinite distance limit is either:



OR



1. Decompactification limit

$$m_{\text{tower}} \simeq \frac{1}{R} \ll \sqrt{T_{\text{string}}}$$

2. Emergent string limit

$$m_{\text{tower}} \simeq \sqrt{T_{\text{string}}}$$

(See Daniel's talk)

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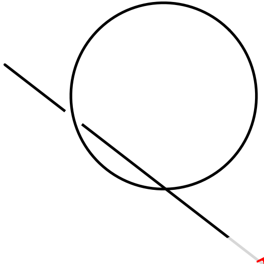

OR

…

$T_{\text{string}} \ll M_d^2$ in both cases

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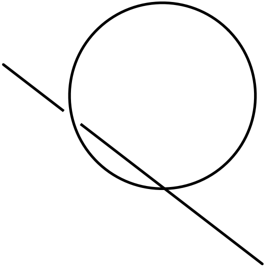

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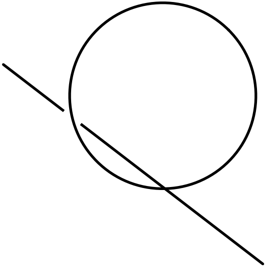

1. Decompactification limit

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Suppose that

$$T_{\text{string}} \sim M_D^2$$

then since $(\text{with } k = D - d)$

$$M_d^{d-2} \sim R^k M_D^{D-2}$$

$$\Longrightarrow$$

$$T_{\text{string}} \sim \frac{M_d^2}{(M_d R)^{\frac{2k}{D-2}}}$$

Emergent String Conjecture (Lee, Lerche, Weigand '19)

Predicts strings satisfying $T_{\text{string}}/M_d \to 0$ in **every** asymptotic limit, except for rare case of decompactification to a theory without strings (e.g., 11d M-theory)

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Test this prediction in vector multiplet moduli space of 5d $\mathcal{N} = 1$ theories (arising from M-theory on a CY3)

(**NOTE**: Overall volume is hypermultiplet.)

5d N=1 SUGRA

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\mathfrak{g}_{ij}(\phi)\partial\phi^i\cdot\partial\phi^j\right) - \frac{1}{2g_5^2}\int a_{IJ}(\phi)F^I\wedge\star F^J$$
$$+ \frac{1}{6(2\pi)^2}\int C_{IJK}A^I\wedge F^J\wedge F^K,$$

$$g_5^2 = (2\pi)^{4/3}(2\kappa_5^2)^{1/3}$$
$$I = 0, 1, \ldots, n$$
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$$\mathcal{F}[Y(\phi)] = 1 \qquad \mathfrak{g}_{ij}(\phi) = a_{IJ}(\phi)\partial_iY^I\partial_jY^J$$
$$a_{IJ} = \mathcal{F}_{IJ} - \mathcal{F}_I\mathcal{F}_J \qquad C_{IJK} = \mathcal{F}_{IJK}$$

$$\mathcal{F}_I = \mathcal{F}_{,I}, \quad \mathcal{F}_{IJ} = \mathcal{F}_{,IJ}, \quad \ldots$$

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BPS particle bound

$$m(\phi) \geq \frac{g_5}{\sqrt{2}\kappa_5}|\zeta(\phi)| = \frac{g_5}{\sqrt{2}\kappa_5}|q_I Y^I(\phi)|$$

BPS string bound ($\tilde{g}_5 = 2\pi/g_5$)

$$T(\phi) \geq \frac{\tilde{g}_5}{\sqrt{2}\kappa_5}|\tilde{\zeta}(\phi)| = \frac{\tilde{g}_5}{\sqrt{2}\kappa_5}|\tilde{q}^I\mathcal{F}_I(\phi)|$$

Homogeneous & dual coordinates

Convenient to **projectivize**: $Y^I \cong \lambda Y^I$

$\mathcal{F}[Y] \neq 0$ has weight 3

$$h_{IJ} = \frac{1}{\mathcal{F}^2}\mathcal{F}_I\mathcal{F}_J - \frac{1}{\mathcal{F}}\mathcal{F}_{IJ}$$

pos. definite
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weight –2

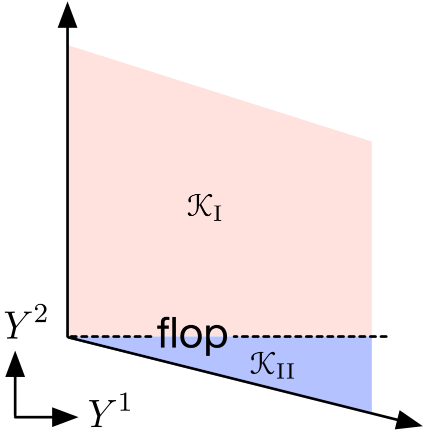
$$\tilde{Y}_I = \frac{1}{\mathcal{F}}\mathcal{F}_I$$

are the **dual coordinates**
weight –1

$$Y^I \leftrightarrow \tilde{Y}_I$$

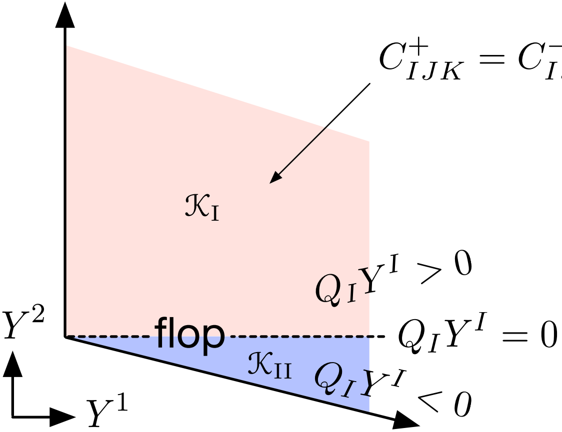
the map between coords and dual coords is bijective in the **interior** of the moduli space

Cones and the Moduli Space



Extended Kähler cone $\mathcal{K}$

Cones and the Moduli Space



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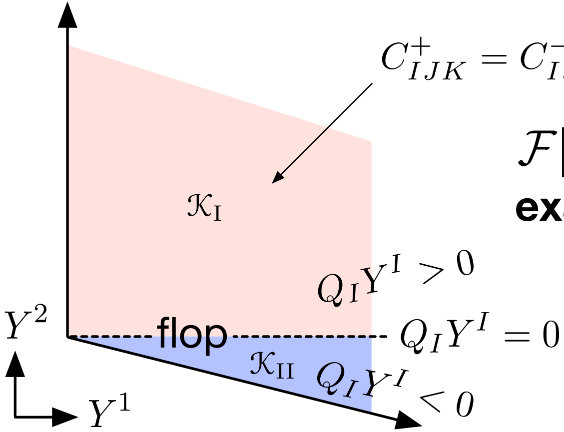

$\mathcal{F}[Y]$ is otherwise **exact**! (unlike in 4d)

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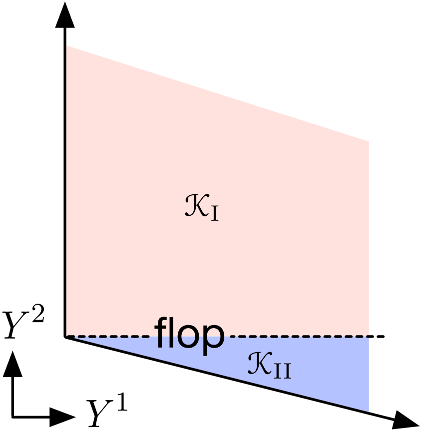

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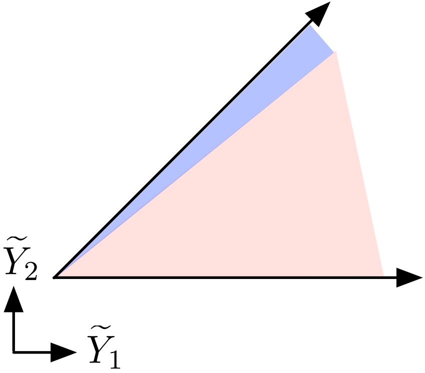
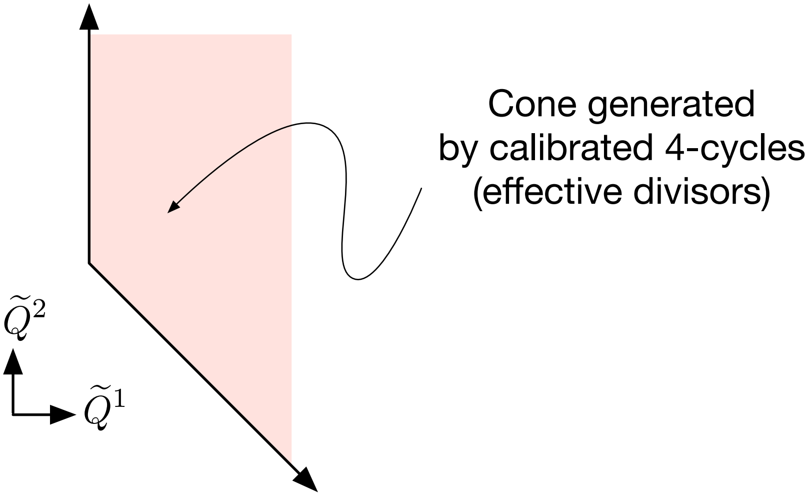


Dual coordinate cone $\widetilde{\mathcal{K}}$

$$\widetilde{\mathcal{K}} \subseteq \mathcal{K}^*$$

Strings and the Effective Cone



Effective cone $\mathcal{E}$

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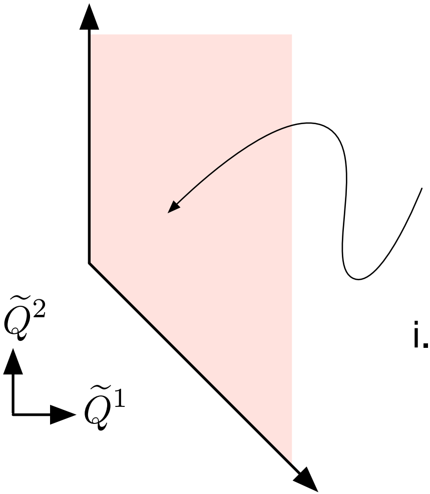
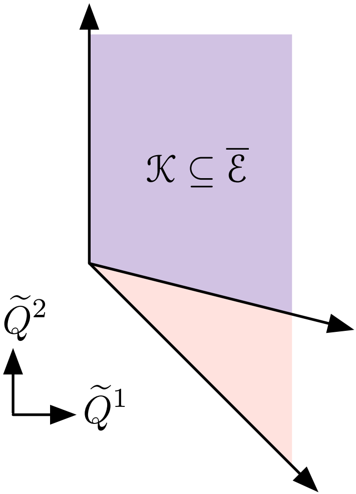


Effective cone $\mathcal{E}$

Cone generated by calibrated 4-cycles (effective divisors)

i.e., the cone generated by the **BPS strings** (from wrapped M5s)

Strings and the Effective Cone



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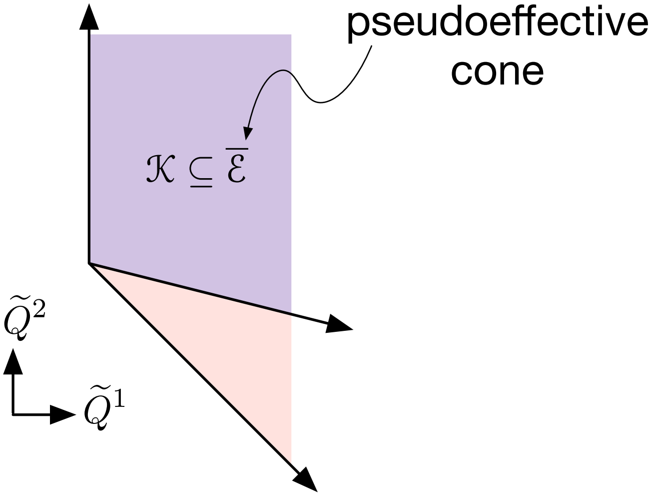

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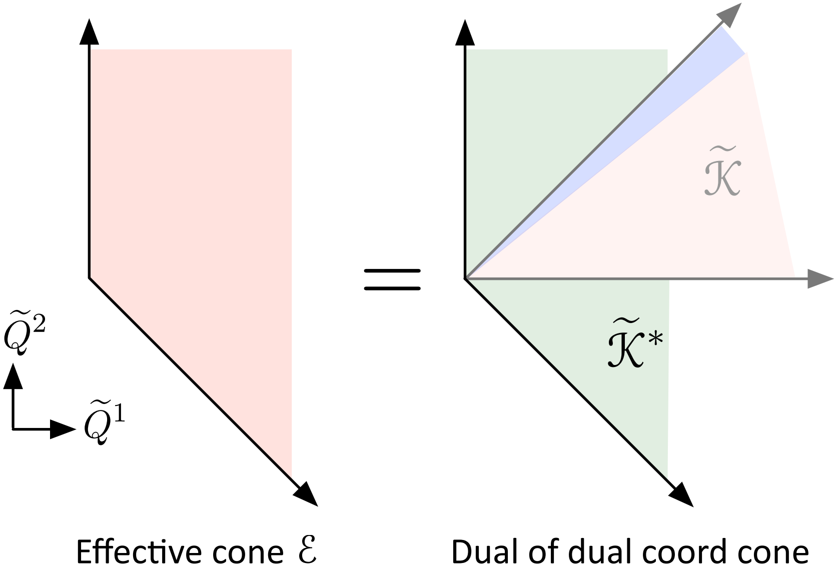

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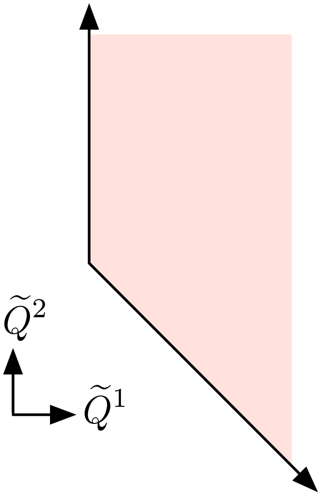
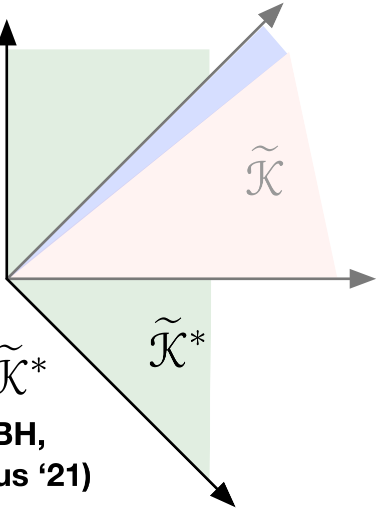


Effective cone $\mathcal{E}$

$$=$$

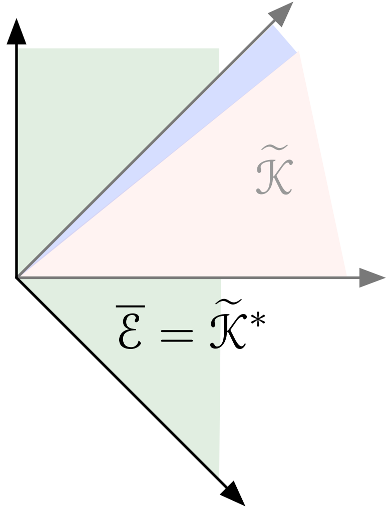
$$\overline{\mathcal{E}} = \widetilde{\mathcal{K}}^*$$

(Alim, BH, Rudelius '21)



Dual of dual coord cone

Tensionless strings at boundaries?

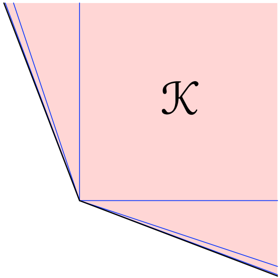


Naively sufficient to ensure that **some** BPS string becomes tensionless at **every** boundary of the moduli space (infinite distance or not)

…**if** the effective cone is closed

$$\mathcal{E} = \overline{\mathcal{E}}$$

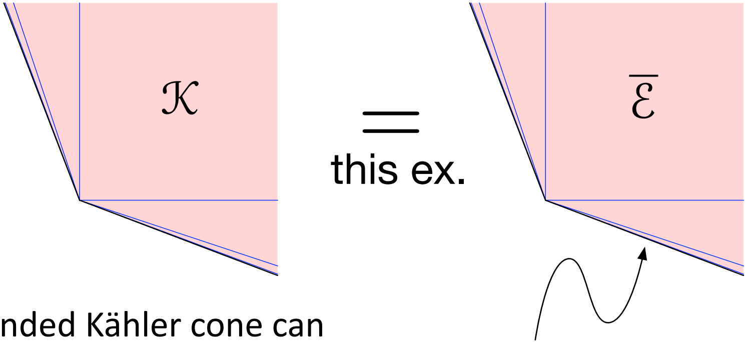
Periodic Boundaries



Extended Kähler cone can contain infinitely many flops*

*Thanks to C. Brodie, A. Constantin, A. Lukas, F. Ruehle for discussions

Periodic Boundaries



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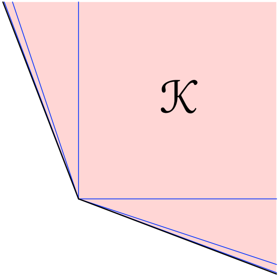
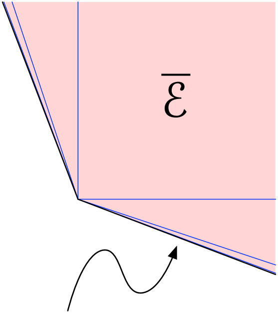
Boundaries have irrational slopes!

Periodic Boundaries



$=$ this ex.



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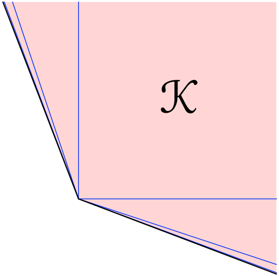

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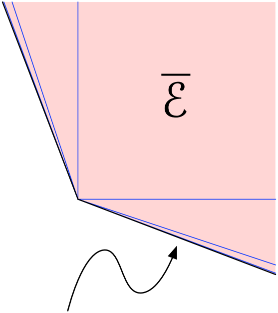

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$$\mathcal{E} \subseteq \mathrm{Hull}(\overline{\mathcal{E}} \cap \mathbb{Q}^k)$$

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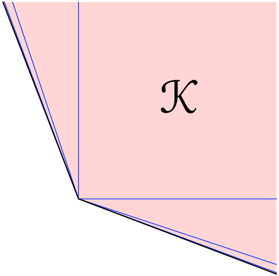

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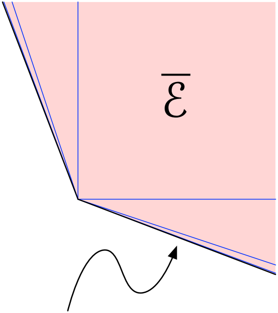

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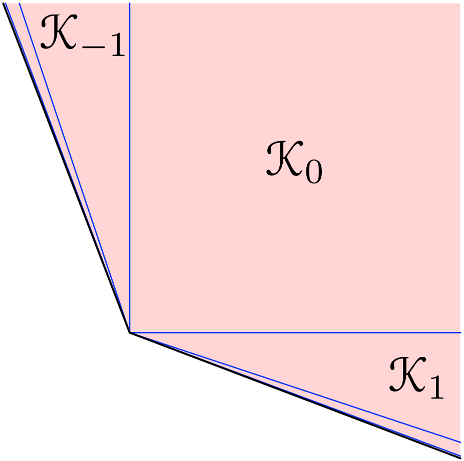
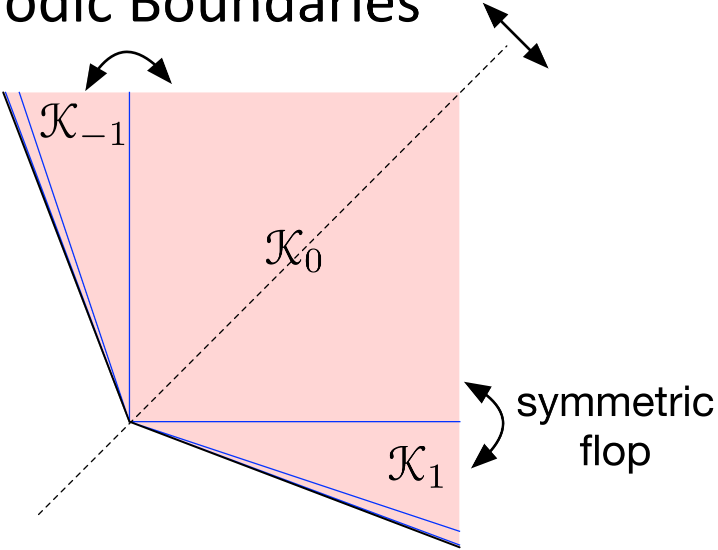


$$\cdots \cong \mathcal{K}_{-1} \cong \mathcal{K}_0 \cong \mathcal{K}_1 \cong \cdots$$

Periodic Boundaries



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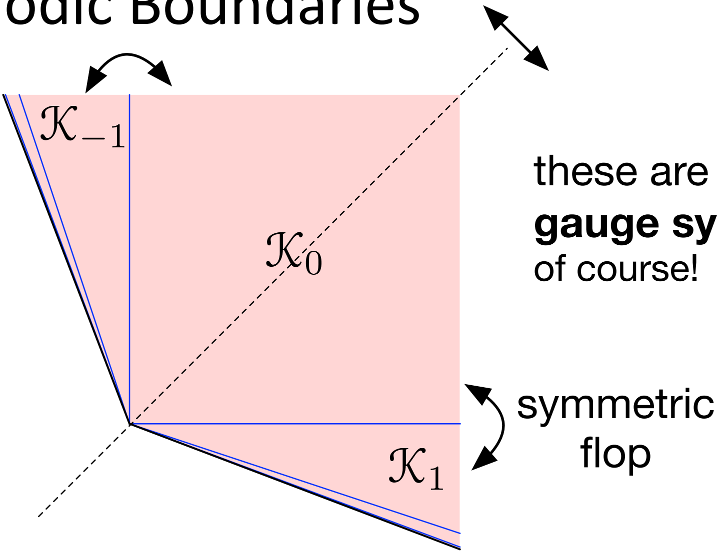

these are all **gauge symms** of course!

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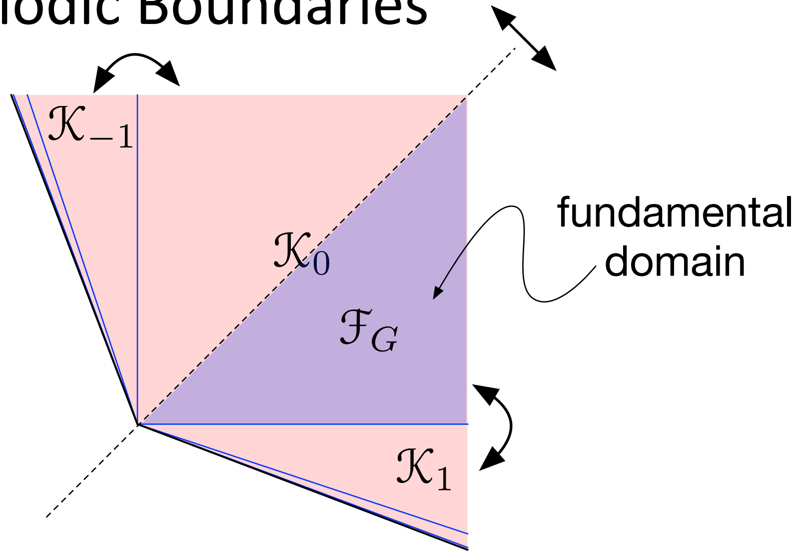

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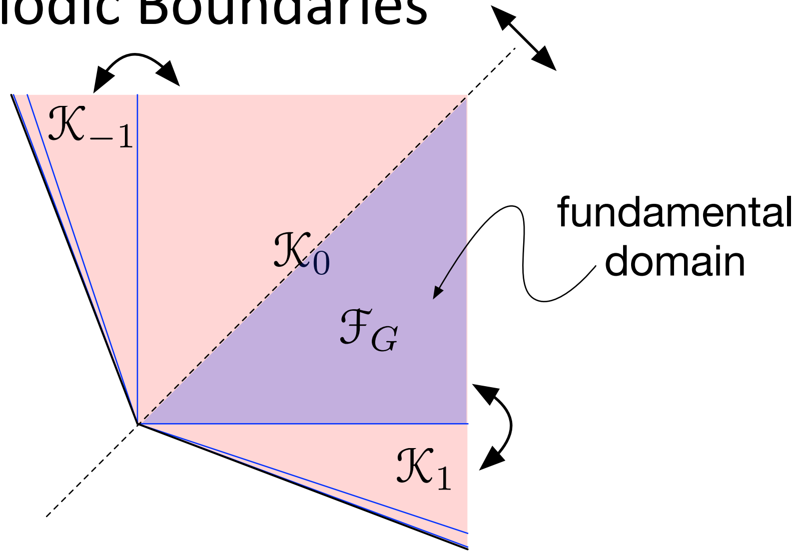

Moduli space has no infinite dist boundary!

The (birational) cone conjecture

There exists a **rational polyhedral** fund. domain $\mathcal{F}_G$ for the automorphism grp G acting on $\mathcal{K}^+$

Morrison '93, '94
Kawamata '97

(See also Fabian's talk)

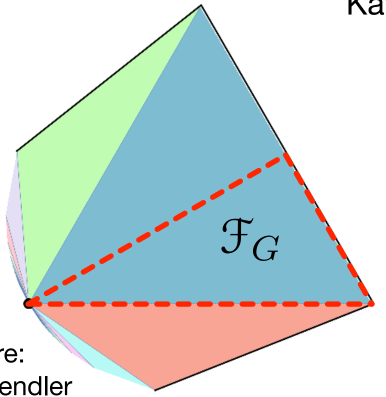

Figure:
N. Gendler

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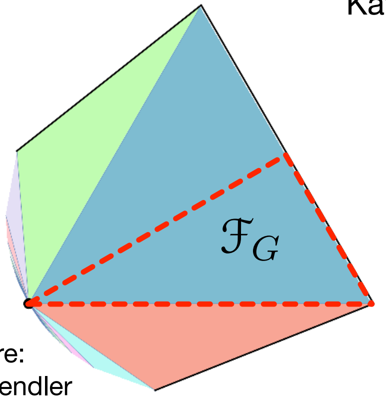

Figure:
N. Gendler

$\mathcal{F}_G$ with appr. boundary idents is the **physical** moduli space

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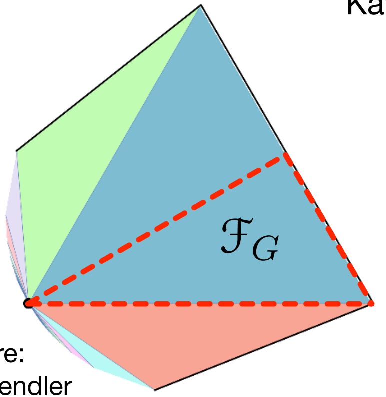

Figure:
N. Gendler

$\mathcal{F}_G$ with appr. boundary idents is the **physical** moduli space

$$\Longrightarrow$$

B'dary points outside $\mathcal{K}^+$ are **unphysical** and **unreachable**

The WGC for BPS Strings

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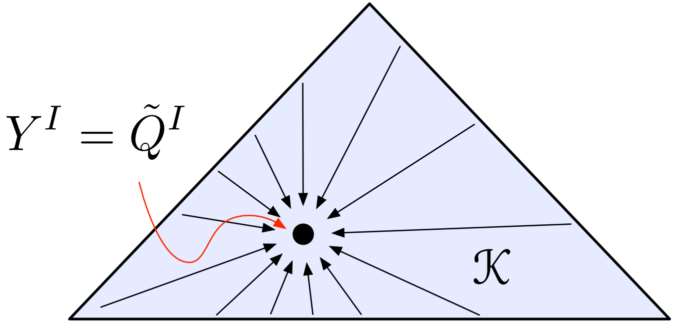
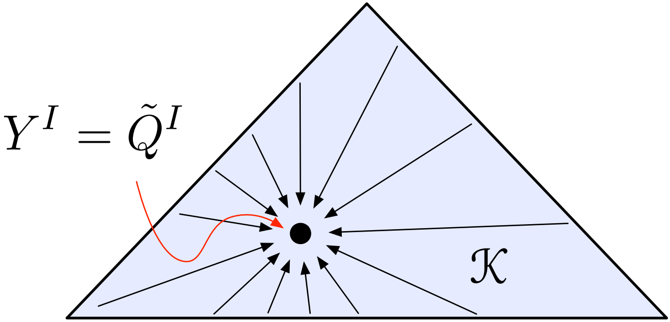


BPS black strings exist throughout $\mathcal{K}^+$

(c.f., Alim, BH, Rudelius '21)

The WGC for BPS Strings

The charges of the BPS strings should span the cone where BPS = Black String Extremal



$\Longrightarrow \mathcal{K}^+ \subseteq \mathcal{E}$ to satisfy WGC for BPS strings!

(nontrivial math conjecture)

Tensionless strings at boundaries?

Cone conjecture: **physical** boundaries lie inside $\mathcal{K}^+$

Either:

1. $\mathcal{F} \to 0$ as $Y^I \to Y^I_*$ $\Longleftrightarrow$ infinite distance

2. $\mathcal{F} \neq 0$ as $Y^I \to Y^I_*$ $\Longleftrightarrow$ **finite** distance

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Cone conjecture: **physical** boundaries lie inside $\mathcal{K}^+$

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   *(In fact, there are tensionless strings **anyway**, due to a shrinking divisor* $\Longrightarrow \mathcal{E} = \mathcal{E}^+$*)*

Sublattice WGC for strings?

Since $\mathcal{F}_G$ finitely generated, cone conjecture + string WGC imply:

There exists $k \in \mathbb{Z}$ such that $k\Gamma_{\mathrm{string}} \cap \mathcal{K}^+$ is generated by non-negative integer LCs of the BPS string charges

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required to satisfy particle sublattice WGC after dim. red.

Infinite towers?

To **ensure** the presence of infinite towers required by the SDC, we need by analogy:

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Realizes idea from Gendler, Valenzuela '20
that tower WGC $\Longrightarrow$ SDC

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If so, the reasoning of BH, Rudelius '20 applies: $g \to 0$ iff we approach inf. distance boundary

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[Note: if $\beta_i \neq 0$ then $\tilde{Q}^I \propto \delta^{I,0}$ is a Kollar divisor

($Y^I_* \in \mathcal{K}^+ \subseteq \mathcal{E}$ as prev. argued)

Signals elliptic fibration of 6d F-theory model!]

Summary

There is a rich interplay between the SDC, emergent strings, the WGC, no global symmetries, and finiteness in 5d $\mathcal{N} = 1$ theories and some existing and novel conjectures about the “tameness” of certain geometric cones